SPHERICAL NEAR-FIELD MEASUREMENTS: GOING OFF GRID

Randal H. Direen, Ph.D., Harry G. Direen, Ph.D
Direentech, Inc.
2750 Old North Gate Rd.
Colorado Springs, CO 80921USA

James D. Huff, Carl W. Sirles, John C. Mantovani
The Howland Company, Inc.
4540 Atwater Court, Suite 107 Buford, GA 30518 USA

ABSTRACT

The speed of spherical near-field scanning is increased significantly when measurements are not restricted to standard measurement locations, i.e., the locations that are equidistant in theta and in phi. Measurement positions can be chosen so that mechanical positioners perform scans with a continuous motion; this will decrease the time it takes to acquire data for near-field measurements. The issue then becomes transforming the data acquired with non-uniform spacing. This paper describes the development of a spherical near-field to far-field transform that can efficiently process data acquired on a non-uniform grid.

Keywords: Spherical Near-Field Measurements, uniform-grid, spiral scan, fast algorithms

1.0 Introduction

When scanning over a spherical surface, near-field measurement points are nearly always restricted to locations that are equidistant in both the theta and phi coordinates[1]. (When data are collected at locations equidistant in both coordinates, we call this set of locations the “uniform-grid.”) To accumulate data on the uniform-grid, mechanical positioners typically scan one axis and step the other. Consequently, positioners are forced to start and stop many times throughout a near-field measurement. If near-field measurements could be made without these constant stops the speed of a measurement would increase significantly.

Consider the spiral scan path of Figure 1. This path is scanned with mechanical positioners in a single motion without stopping between measurement points. Each of the points located at the marks along the spiral path are collected as the positioner continuously scans over the sphere. Clearly, a scan of this kind will be many times

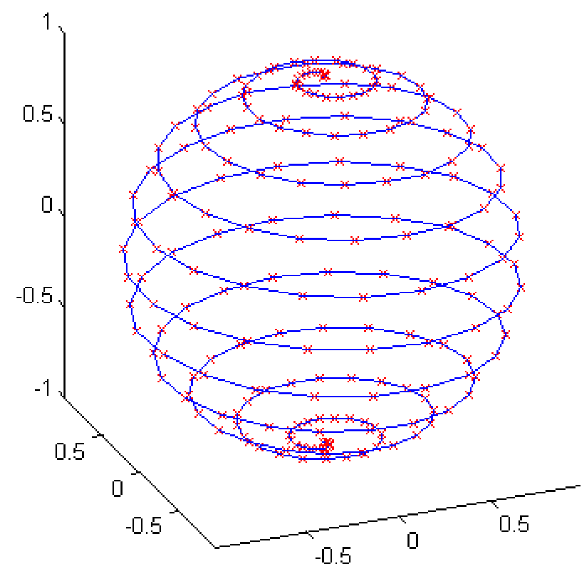

Figure 1, Spiral scanning path. Data are collected at the marks along the path.

faster than one that continually stops throughout the measurement. Considering the benefits, the reader may wonder why spiral scanning is not more common in near-field measurements.

Historically, near-field software has benefitted greatly from fast algorithms such as the FFT (Fast Fourier Transform). Even for relatively small antennas, it was not practical in the early days of near-field measurements to perform near-to-far-field transformations without special algorithms (processing data could take days or weeks without special algorithms). Today, data are restricted to a uniform-grid because these early algorithms required it.

The authors of this paper have developed a spherical near-to-far-field transform algorithm that is not restricted to specific measurement locations. The near-to-far-field transformation can be calculated quickly even when data are collected, for example, using the spiral scan of Figure 1. With this new algorithm, the time it takes to process near-field data, even for moderately large antenna

[1] In other scanning geometries near-field measurements are also usually taken at points that are equidistant in the two scanning axes. In this paper we only consider the spherical case.

problems, is not significantly different from the earlier software packages (transformations are performed in a matter of seconds on modern computers).

After years of measuring near-field data on the uniform-grid, it may take the reader some convincing to see that scanning off of the uniform-grid is a viable thing to do. In this paper, we show that sampling on the uniform-grid was only ever important for the purpose of speeding up certain near-to-far-field transform algorithms.

We provide a small numerical example showing how the speed of our processing algorithm compares to the speed of the standard NIST algorithm. We compare this speed to brute force methods.

2.0 Theory

In spherical near-field measurements, the response $\mathbf{w}(\hat{\mathbf{r}}_k)$ [1] of the probe to the near-field is recorded over the surface of the measurement sphere at L independent locations $\hat{\mathbf{r}}_k = (\theta_k, \phi_k)$. The probe response $\mathbf{w}(\hat{\mathbf{r}}_k)$ is a vector quantity having components in both the theta and phi directions. It is necessary in standard near-field transform to approximate the probe response by a supposition of vector spherical harmonics:

$$\mathbf{w}(\hat{\mathbf{r}}_k) \approx \sum_{n=1}^{N} \sum_{m=-n}^{n} a_{nm}^{(1)} \mathbf{X}_{nm}(\hat{\mathbf{r}}_k) + a_{nm}^{(2)} \mathbf{Y}_{nm}(\hat{\mathbf{r}}_k) \quad (0.1)$$

Where $\mathbf{X}_{nm}(\hat{\mathbf{r}})$ and $\mathbf{Y}_{nm}(\hat{\mathbf{r}}) = i\hat{\mathbf{r}} \times \mathbf{X}_{nm}(\hat{\mathbf{r}})$ are vector spherical harmonics [2] and $a_{nm}^{(1,2)}$ are the corresponding coefficients. For an antenna that fits within a minimum sphere [1] of radius a, we choose $N \approx 2\pi a / \lambda$ where λ is the wavelength being measured by the receiver. The far-field pattern can then be calculated after additional processing of the coefficients $a_{nm}^{(1,2)}$.

Calculating the $2N(N+2)$ unknown coefficients $a_{nm}^{(1,2)}$ in equation (0.1) is the greatest bottleneck in the near-to-far-field calculation. The coefficients can be calculated using brute force methods if $L = 2N(N+2)$ independent locations $\hat{\mathbf{r}}_k$ are sampled and a numerical method such as Gaussian elimination [3] is applied. Unfortunately, Gaussian elimination requires $O(N^6)$ operations and for typical problem sizes $(10^2 < N < 10^3)$ this is generally impractical for even modern computers.

Equation (0.1) represents a linear system of equations, and provided the equations are linearly independent, the system can be solved using standard techniques. Solving for coefficients does not require data to be taken on the uniform-grid, and can in fact be taken anywhere. (Numerically, however, we must be concerned with the conditioning number [3] of the linear operator that takes the coefficients $a_{nm}^{(1,2)}$ to the measured data $\mathbf{w}(\hat{\mathbf{r}}_k)$.)

The fast algorithms used in most near-field software (including NIST based software) require the sample locations $\hat{\mathbf{r}}_k$ to be on a uniform-grid. These algorithms [4] calculate the coefficients $a_{nm}^{(1,2)}$ in $O(N^3)$ operations. This significantly reduces the time it takes to calculate the coefficients so that problem sizes of $10^2 < N < 10^3$ can be calculated in a matter of seconds (as opposed to hours, and potentially days, without fast algorithms). The uniform-grid requirement forces mechanical positioners to make constant stops, slowing the measurement process down significantly.

The authors of this paper have developed a new near-to-far-field transform algorithm that calculates the coefficients $a_{nm}^{(1,2)}$ in $O(N^3)$ operations, but does not require data to be on the uniform-grid. Now measurements can be taken on scan paths such as the spiral of Figure 1 and do so very quickly with mechanical scanners that sweep scans in a continuous motion. The resulting far-field calculated with this new algorithm is identical to the results calculated with the NIST based algorithm since it is capable of retrieving the exact same coefficients $a_{nm}^{(1,2)}$ in the expansion of Equation(0.1). (For the interested reader, this new software relies on work developed by Dutt and Rocklin [5], and Wittmann, Alpert, and Francis [6,7].)

3.0 Numeric Example

To shed light on the importance of fast algorithms for the processing of near-to-far-field transformations, simulations were run using the standard NIST software and software using the new algorithm. This example only shows the speed in which it takes to invert Equation (0.1) and not how accurate the results are. Two data sets were taken, the first on a uniform grid and the second on a spiral grid as shown in Figure 1. We collected enough samples on both the uniform-grid and on the spiral so that we could accurately recover the necessary coefficients $a_{nm}^{(1,2)}$ (determining the number of samples requires an understanding of the condition number of the linear set of equations in Equation (0.1)).

Table 1 shows the time it took to compute coefficients $a_{nm}^{(1,2)}$ using three different methods on a standard notebook computer for different numbers of sample points. The first method of computing the coefficients uses standard NIST software. The second method uses software with the new algorithm, and the third method uses the brute-force Gaussian Elimination method. While the software with the new algorithm takes approximately 5 times longer to compute the coefficients as compared to the NIST software, it is still only a few seconds. The measurement time gained by using a non-uniform grid with a spiral scan will be far more significant.

Table 1 Computation Time (seconds) to Compute Coefficients

N	Software Using the NIST Algorithm	Software Using the New Algorithm	Gaussian Elimination
50	0.49	2.3	220
70	0.59	2.9	1700
90	0.72	3.8	7600
110	0.88	4.8	25000

4.0 Conclusion and Future Work

We have developed an algorithm that processes near-field measurements taken off of the uniform-grid (not only for spherical, but for planar and cylindrical as well). It is now possible to take measurements much faster without having to stop mechanical scanners many times throughout a scan.

We have not answered obvious questions in this short paper. Future work will show just how much time can be saved by scanning on paths such as the one shown in Figure 1. We have stated that there are no fundamental differences between measuring at locations on the uniform-grid and measuring at locations off of the grid, but we do need to address numerical conditioning issues that affect our decision on how we choose new locations. We would also like to study how our technique compares with the work provided by Bucci et al [8]. Finally, we have given no real details about how we have achieved the numerical speeds presented in this paper. These details will be left for a future paper.

5.0 References

[1] M. H. Francis and R. C. Wittmann, "Near-field scanning measurements: theory and practice," in *Modern Antenna Handbook*, C. A. Balanis, Ed., chapter 19, John Wiley & Sons, New York, NY, USA, 2008.

[2] J. D. Jackson, *Classical Electrodynamics*, John Wiley & Sons, New York, NY, USA, 2nd edition, 1975.

[3] G. H. Golub and C. F. Van Loan, *Matrix Computations*, The Johns Hopkins University Press, Baltimore, Md, USA, 1989.

[4] Hansen, J. E. *Spherical Near-field Antenna Measurements*. London, U.K.: P. Peregrinus on Behalf of the Institution of Electrical Engineers, 1988. Print.

[5] A. Dutt and V. Rokhlin, "Fast Fourier transforms for nonequispaced data," *SIAM J. Scientific Comput.*, vol. 14, pp. 1369–1393, Nov. 1993.

[6] R. C. Wittmann, B. K. Alpert, and M. H. Francis, "Near-field, spherical-scanning antenna measurements with nonideal probe locations," *IEEE Transactions on Antennas and Propagation*, vol. 52, no. 8, pp. 2184–2186, 2004.

[7] R. C. Wittmann, B. K. Alpert, and M. H. Francis, "Planar near-field antenna measurements using nonideal measurement locations," in *Proc. Antenna Measurement Tech. Assoc.*, Seattle, WA, Sept. 30–Oct. 6, 1996, pp. 74–77.

[8] Bucci, O. M., F. D'Agostino, C. Gennarelli, G. Riccio, and C. Savarese, "NF-FF transformation with spherical spiral scanning," *IEEE Antennas Wireless Propagat. Lett.*, Vol. 2, 263-266, 2003.

9. Acknowledgements

The authors thank Ron Wittmann for many helpful discussions.